$$-2x+2y-2z=0 \qquad \frac{x}{-3+1} = \frac{y}{1+3} = \frac{z}{-1-1}$$

$$x+y+z=0$$

$$x-y-3=0 \qquad \frac{x}{-2} = \frac{y}{4} = \frac{z}{-2}$$

When $\lambda = \sqrt{3}i$

$$\begin{vmatrix} 1-\sqrt{3}i & 2 & -2 \\ 1 & 2-\sqrt{3}i & 1 \\ 1 & -1 & -\sqrt{3}i \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$$(1-\sqrt{3}i)x + 2y - 2z = 0$$

$$x + (2-\sqrt{3}i)y + z = 0$$

$$x - y - \sqrt{3}iz = 0$$

$$\frac{x}{-2\sqrt{3}i-3+1} = \frac{y}{1+\sqrt{3}i} = \frac{z}{-3+\sqrt{3}i}$$

Now when $\lambda = -\sqrt{3}i$

$$\begin{bmatrix} 1+\sqrt{3}i & 2 & -2 \\ 1 & 2+\sqrt{3}i & i \\ 1 & -1 & \sqrt{3}i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$(1+\sqrt{3}i)x + 2y - 2z = 0$$

$$x + (2+\sqrt{3}i)y + z = 0$$

$$x - y + \sqrt{3}iz = 0$$

$$\frac{x}{2\sqrt{3}i-3-1} = \frac{y}{1-\sqrt{3}i} = \frac{z}{-1-2-\sqrt{3}i}$$

$$\frac{x}{2\sqrt{3}i-1} = \frac{y}{1-\sqrt{3}i} = \frac{z}{-3-\sqrt{3}i} \qquad P =$$

$$P = \begin{bmatrix} -2 & -2\sqrt{3}i-2 & 2\sqrt{3}i \\ 4 & 1+\sqrt{3}i & 1-\sqrt{3}i \\ -2 & -3+\sqrt{3}i & -3-\sqrt{3}i \end{bmatrix}$$

largest eigen value & eigen vector

Power Method for eigen value & eigen vector

This is an ittirated method which is used to calculate the eigen value & eigen vector of a Matric consisting 'n' order.

let $X = c_1x_1 + c_2x_2 + c_3x_3 + \cdots$

let the eigen value of corresponding coordinates will be $\lambda_1, \lambda_2, \lambda_3 \cdots$ Then

$$AX = c_1AX_1 + c_2AX_2 + c_3AX_3 + \cdots$$

Then from the condⁿ of eigen value we have know that $AX = \lambda x$

Then $AX = c_1\lambda_1 x_1 + c_2\lambda_2 x_2 + c_3\lambda_3 x_3 + \cdots$

Illy $A^2X = c_1\lambda_1^2 x_1 + c_2\lambda_2^2 x_2 + c_3\lambda_3^2 x_3 + \cdots$

$$\vdots$$

$$A^mX = c_1\lambda_1^m x_1 + c_2\lambda_2^m x_2 + c_3\lambda_3^m x_3 + \cdots$$

Now acc. to the power method the value component can be expressed in the form of

$$\boxed{Y^i = AX^{i-1}}$$

Corresponding eigen value $AX = \lambda x$

This is the sufficient condⁿ of Power method which give the eigen value & eigen vector let us consider a matrix for the eigen vectors such that

$$P = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$$

Then

$$AP = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1x_1 + b_1y_1 + c_1z_1 & a_1x_2 + b_1y_2 + c_1z_2 & a_1x_3 + b_1y_3 + c_1z_3 \\ a_2x_1 + b_2y_1 + c_2z_1 & a_2x_2 + b_2y_2 + c_2z_2 & a_2x_3 + b_2y_3 + c_2z_3 \\ a_3x_1 + b_3y_1 + c_3z_1 & a_3x_2 + b_3y_2 + c_3z_2 & a_3x_3 + b_3y_3 + c_3z_3 \end{bmatrix}$$

on using eq^n (1), (2) & (3) we get

$$\begin{bmatrix} \lambda_1 x_1 & \lambda_2 x_2 & \lambda_3 x_3 \\ \lambda_1 y_1 & \lambda_2 y_2 & \lambda_3 y_3 \\ \lambda_1 z_1 & \lambda_2 z_2 & \lambda_3 z_3 \end{bmatrix}$$

$$AP = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Then

$$AP = PD$$

where $D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$ is known as Diagonal matrix

also multiplying by P^{-1} we get

$$P^{-1}AP = P^{-1}PD$$

Then $\boxed{P^{-1}AP = D}$

This eq^n represent the diagonalization of the matrix.

Ques: find the diagonal ~~elements~~ elements of the matrix.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

Sol^n

$$A - \lambda I = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & -1 & 0-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[1] - 2[-1] - 2[-1-2+\lambda] = 0$$

$$(1-\lambda) + 2 - 2(-3+\lambda) = 0$$

$$(1-\lambda) + 2 + 6 - 2\lambda = 0$$

$$(1-\lambda) + 8 - 2\lambda = 0$$

$$-3\lambda + 9 = 0$$

$$3\lambda = 9 \Rightarrow \lambda = 3$$

$(1-\lambda(-2+\lambda^2 \ldots)$
$-1 - 2[-\lambda+1] - 2(-1 \ldots)$

$\lambda^3 - 3\lambda^2 + 3$

$(\lambda-3)(\lambda^2+3) = 0$

$\lambda = 3, \pm\sqrt{3}i$

Now eigen vector of above eq^n can be written as when $\lambda = 3$

$$\begin{bmatrix} -2 & 2 & -2 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} -2 & 2 & -2 \\ 1 & 2 & 1 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

of the matrix.
In this method the eigen value remain in its max form.

Ques Find the eigen value & eigen vectors of the matrix $A = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$

Sol^n The corresponding eigen vector of above matrix will be $X = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Then $y^1 = AX = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$= 4\begin{bmatrix} 1 \\ 0.25 \end{bmatrix}$

we also know that

$AX = \lambda X$

Then on comparing we get $\lambda_1 = 4$ & $X_1 = \begin{bmatrix} 1 \\ 0.25 \end{bmatrix}$

Now the II^nd ittiration can be calculated as

$y^2 = AX^1 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 4.25 \\ 1.75 \end{bmatrix}$

$= 1.25\begin{bmatrix} 1 \\ 0.41 \end{bmatrix}$

$\lambda^2 = 4.25$ & $X^2 = [0.41]$

llly $y^3 = AX^2 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.41 \end{bmatrix} = \begin{bmatrix} 4.41 \\ 2.23 \end{bmatrix}$

$= 4.41\begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$

$\lambda^3 = 4.41$ & $X^3 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$

llly $y^4 = AX^3 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 4.5 \\ 2.5 \end{bmatrix}$.

$= 4.5\begin{bmatrix} 1 \\ 0.55 \end{bmatrix}$

$\lambda_4 = 4.5$ & $X^4 = \begin{bmatrix} 1 \\ 0.55 \end{bmatrix}$

→ llly $y^5 = AX^4 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 4.55 \\ 2.65 \end{bmatrix}$

$= 4.55\begin{bmatrix} 1 \\ 0.58 \end{bmatrix}$

$\lambda_5 = 4.55$ & $X^5 = \begin{bmatrix} 1 \\ 0.58 \end{bmatrix}$

→ Now $y^6 = AX^5 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.58 \end{bmatrix} = \begin{bmatrix} 4.58 \\ 2.74 \end{bmatrix}$

$= 4.58\begin{bmatrix} 1 \\ 0.59 \end{bmatrix}$, $\lambda_6 = 4.58$; $X_6 = \begin{bmatrix} 1 \\ 0.59 \end{bmatrix}$

→ $y^7 = AX^6 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.59 \end{bmatrix}$

$= \begin{bmatrix} 4.59 \\ 2.77 \end{bmatrix} = 4.59\begin{bmatrix} 1 \\ 0.60 \end{bmatrix}$

$\lambda_7 = 4.59$ & $X_7 = \begin{bmatrix} 1 \\ 0.60 \end{bmatrix}$

→ $y^8 = AX^7 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.60 \end{bmatrix} = \begin{bmatrix} 4.60 \\ 2.00 \end{bmatrix}$

$$= 4.60 \begin{bmatrix} 1 \\ 0.60 \end{bmatrix}$$

$$\lambda_8 = 4.60, \quad X_8^8 = \begin{bmatrix} 1 \\ 0.60 \end{bmatrix}$$

IIIly $X^8 = \begin{bmatrix} 1 \\ 0.60 \end{bmatrix}$

from above its clear that $X^8 - X^7 = 0$

Hence $\begin{bmatrix} 1 \\ 0.60 \end{bmatrix}$ are the required eigen vectors

& $\lambda = 4.60$ is the required eigen value of given matrix.

Ques $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ & $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$Y^1 = AX = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.85 \\ -0.28 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 7 \ \& \ X^2 = \begin{bmatrix} 0.85 \\ -0.28 \\ 1 \end{bmatrix}$$

→ also $Y^2 = AX^2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.85 \\ -0.28 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.72 \\ 3.12 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.39 \\ 0.44 \\ 1 \end{bmatrix}$

$$\lambda_3 = 7 \ \& \ X^3 = \begin{bmatrix} 0.39 \\ 0.44 \\ 1 \end{bmatrix}$$

→ $Y^3 = AX^3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.39 \\ 0.44 \\ 1 \end{bmatrix} = \begin{bmatrix} 4.27 \\ 0.24 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.61 \\ 0.03 \\ 1 \end{bmatrix}$

$$\lambda_4 = 7 \ \& \ X^4 = \begin{bmatrix} 0.61 \\ 0.03 \\ 1 \end{bmatrix}$$

→ $Y^4 = AX^4 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.61 \\ 0.03 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.67 \\ 1.88 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.52 \\ 0.26 \\ 1 \end{bmatrix}$

$$\lambda_5 = 7 \ \& \ X^5 = \begin{bmatrix} 0.52 \\ 0.26 \\ 1 \end{bmatrix}$$

$$Y^5 = AX^5 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.52 \\ 0.26 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.64 \\ 0.96 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.5 \\ 0.13 \\ 1 \end{bmatrix}$$

$$\lambda_6 = 7 \ \& \ X^6 = \begin{bmatrix} 0.5 \\ 0.13 \\ 1 \end{bmatrix}$$

→ $Y^6 = AX^6 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.13 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.76 \\ 1.48 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.53 \\ 0.21 \\ 1 \end{bmatrix}$

$$\lambda_7 = 7 \ \& \ X^7 = \begin{bmatrix} 0.53 \\ 0.21 \\ 1 \end{bmatrix}$$

→ $Y^7 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.53 \\ 0.21 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.95 \\ 1.16 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.5 \\ 0.16 \\ 1 \end{bmatrix}$

$$\lambda_8 = 7 \ \& \ X^8 = \begin{bmatrix} 0.5 \\ 0.16 \\ 1 \end{bmatrix}$$

→ $Y^8 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.16 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.82 \\ 1.36 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.54 \\ 0.19 \\ 1 \end{bmatrix}$

→ $Y^9 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.54 \\ 0.19 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.92 \\ 1.24 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.56 \\ 0.17 \\ 1 \end{bmatrix}$

→ $Y^{10} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 0.56 \\ 0.17 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.9 \\ 1.32 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 0.55 \\ 0.18 \\ 1 \end{bmatrix}$